Introduction to Game Theory

Ling-Chieh Kung

Department of Information Management National Taiwan University

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Why game theory?

- Industrial engineers and operations researchers optimize the performance of systems.
 - ▶ Many systems have **multiple decision makers** involved.
 - Competing retailers in a market.
 - Entities in a supply chain.
 - ▶ Deliverers and consumers on a food delivery platform.
 - Companies bidding for 5G bandwidth.
 - ▶ Voters and candidates in an election.
- ► To model the interaction among multiple decision makers, game theory helps people design **better mechanisms or policies**.

Introduction

- ► Today we introduce **game-theoretic modeling**.
- ▶ We will introduce **static** and **dynamic** games.
 - ▶ Static games: All players act simultaneously (at the same time).
 - ▶ Dynamic games: Players act sequentially.
- ▶ We will illustrate the **inefficiency** caused by decentralization (lack of cooperation).
 - With the concept of **price of anarchy**.
- We will show how to solve a game, i.e., to predict what players will do in equilibrium.

Road map

► Static games.

Prisoners' dilemma.

- Nash equilibrium.
- Cournot competition.
- ▶ Dynamic games: Backward induction.
- Pricing in a supply chain.

Static games	Dynamic games	Price of anarchy	Equilibrium concepts
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Prisoners' dilemma: story

- ▶ A and B broke into a grocery store and stole some money. Before police officers caught them, they hided those money carefully without leaving any evidence. However, a monitor got their images when they broke the window.
- ► They were kept in two separated rooms. Each of them were offered two choices: **Denial or confession**.
 - If both of them deny the fact of stealing money, they will both get one month in prison.
 - ▶ If one of them confesses while the other one denies, the former will be set free while the latter will get nine months in prison.
 - ▶ If both confesses, they will both get six months in prison.
- They cannot communicate and they must make their choices simultaneously.
- ▶ All they want is to be in prison as short as possible.
- ▶ What will they do?

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Prisoners' dilemma: matrix representation

▶ We may use the following matrix to formulate this "game":

	Player 2		
		Denial	Confession
Player 1	Denial	-1, -1	-9,0
	Confession	0, -9	-6, -6

- ▶ There are two **players**, each has two possible **actions**.
- ▶ For each combination of actions, the two numbers are the utilities of the two players: the first for player 1 and the second for player 2.
- Prisoner 1 thinks:

- "If he denies, I should confess."
- "If he confesses, I should still confess."
- "I see! I should confess anyway!"
- ▶ For prisoner 2, the situation is the same.
- ▶ The **solution** (outcome) of this game is that both will confess.

Equilibrium concepts 00000

Prisoners' dilemma: discussions

- ▶ In this game, confession is said to be a **dominant strategy**.
- ▶ This outcome can be "improved" if they can **cooperate**.
- **Lack of cooperation** can result in a **lose-lose** outcome.
 - Such a situation is **socially inefficient**.
- ▶ We will see more situations similar to the prisoners' dilemma.

Solutions for a game

Is it always possible to solve a game by finding dominant strategies?What are the solutions of the following games?

▶ We need a new solution concept: Nash equilibrium!

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Nash equilibrium: definition

▶ The most fundamental equilibrium concept is the Nash equilibrium:

Definition 1 (Nash equilibrium)

For an n-player game, let S_i be player *i*'s action space and u_i be player *i*'s utility function, i = 1, ..., n. An action profile $(s_1^*, ..., s_n^*)$, $s_i^* \in S_i$, is a (pure-strategy) Nash equilibrium if

$$u_i(s_1^*, \dots, s_{i-1}^*, s_i^*, s_{i+1}^*, \dots, s_n^*) \\\geq u_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*)$$

for all $s_i \in S_i$, i = 1, ..., n.

- Alternatively, $s_i^* \in \underset{s_i \in S_i}{\operatorname{argmax}} \left\{ u_i(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_n^*) \right\}$ for all i.
- ▶ In a Nash equilibrium, no one has an incentive to **unilaterally deviate**.
- ▶ The term "pure-strategy" will be explained later.

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Nash equilibrium: an example

• Consider the following game with no dominant strategy:

	Player 2			
		\mathbf{L}	С	R
Player 1	Т	0,4	4,0	5,3
	М	4,0	0,4	5,3
	В	3, 5	3,5	6, 6

- ▶ What is a Nash equilibrium?
 - ▶ (T, L) is not: Player 1 will deviate to M or B.
 - ▶ (T, C) is not: Player 2 will deviate to L or R.
 - ▶ (B, R) is: No one will unilaterally deviate.
 - Any other Nash equilibrium?
- ▶ Why a Nash equilibrium is an "outcome"?
 - Imagine that they takes turns to make decisions until no one wants to move. What will be the outcome?

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Nash equilibrium: More examples

▶ Is there any Nash equilibrium of the prisoners' dilemma?

▶ How about the following two games?



Existence of a Nash equilibrium

	Η	Т
Н	1, -1	-1, 1
Т	-1, 1	1, -1

- The last game does not have a "pure-strategy" Nash equilibrium.
- What if we allow randomized (mixed) strategy?
- In 1950, John Nash proved the following theorem regarding the existence of "mixed-strategy" Nash equilibrium:

Proposition 1

For a static game, if the number of players is finite and the action spaces are all finite, there exists at least one mixed-strategy Nash equilibrium.

- ▶ This is a sufficient condition. Is it necessary?
- In most business applications of Game Theory, people focus only on pure-strategy Nash equilibria.

Road map

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- Prisoners' dilemma.
- Nash equilibrium.
- Cournot competition.
- ▶ Dynamic games: Backward induction.
- Pricing in a supply chain.

Cournot Competition

- In 1838, Antoine Cournot introduced the following quantity competition between two retailers.
- Let q_i be the production quantity of firm i, i = 1, 2.
- ▶ Let P(Q) = a Q be the market-clearing price for an aggregate demand $Q = q_1 + q_2$.
- Unit production cost of both firms is c < a.
- Each firm wants to maximize its profit.
- Our questions are:
 - ▶ In this environment, what will these two firms do?
 - ► Is the outcome satisfactory?
 - What is the difference between duopoly and monopoly (i.e., decentralization and integration)?

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Cournot Competition

- ▶ Players: 1 and 2.
- Action spaces: $S_i = [0, \infty)$ for i = 1, 2.
- ▶ Utility functions:

$$u_1(q_1, q_2) = q_1 \Big[a - (q_1 + q_2) - c \Big]$$
 and
 $u_2(q_1, q_2) = q_2 \Big[a - (q_1 + q_2) - c \Big].$

As for an outcome, we look for a Nash equilibrium.
If (q₁^{*}, q₂^{*}) is a Nash equilibrium, it must solve

$$\begin{aligned} q_1^* &\in \underset{q_1 \in [0,\infty)}{\operatorname{argmax}} \ u_1(q_1, q_2^*) = \underset{q_1 \in [0,\infty)}{\operatorname{argmax}} \ q_1 \Big[a - (q_1 + q_2^*) - c \Big] \text{ and} \\ q_2^* &\in \underset{q_2 \in [0,\infty)}{\operatorname{argmax}} \ u_2(q_1^*, q_2) = \underset{q_2 \in [0,\infty)}{\operatorname{argmax}} \ q_2 \Big[a - (q_1^* + q_2) - c \Big]. \end{aligned}$$

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Solving the Cournot competition

- ► For firm 1, we first see that the objective function is strictly concave: • $u'_1(q_1, q_2^*) = a - q_1 - q_2^* - c - q_1$. $u''_1(q_1, q_2^*) = -2 < 0$.
- ▶ The first-order condition suggests $q_1^* = \frac{1}{2}(a q_2^* c)$ as the **best** response function.
 - If $q_2^* < a c$, q_1^* is optimal for firm 1.
- ▶ Similarly, $q_2^* = \frac{1}{2}(a q_1^* c)$ is firm 2's optimal decision if $q_1^* < a c$.
- ▶ If (q_1^*, q_2^*) is a Nash equilibrium such that $q_i^* < a c$ for i = 1, 2, it must satisfy

$$q_1^* = \frac{1}{2}(a - q_2^* - c)$$
 and $q_2^* = \frac{1}{2}(a - q_1^* - c).$

▶ The unique solution to this system is $q_1^* = q_2^* = \frac{a-c}{3}$.

- Does this solution make sense?
- As $\frac{a-c}{3} < a-c$, this is indeed a Nash equilibrium. It is also unique.

Distortion due to decentralization

- ▶ What is the "cost" of decentralization?
- Suppose the two firms' are integrated together to jointly choose the aggregate production quantity.
- ► They together solve

$$\max_{Q \in [0,\infty)} Q[a - Q - c],$$

whose optimal solution is $Q^* = \frac{a-c}{2}$.

- ► First observation: $Q^* = \frac{a-c}{2} < \frac{2(a-c)}{3} = q_1^* + q_2^*$.
- Why does a firm intend to increase its production quantity under decentralization?

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Inefficiency due to decentralization

- ▶ May these firms improve their profitability with integration?
- \blacktriangleright Under decentralization, firm *i* earns

$$\pi_i^D = \frac{(a-c)}{3} \left[a - \frac{2(a-c)}{3} - c \right] = \left(\frac{a-c}{3} \right) \left(\frac{a-c}{3} \right) = \frac{(a-c)^2}{9}.$$

▶ Under integration, the two firms earn

$$\pi^{C} = \frac{(a-c)}{2} \left[a - \frac{a-c}{2} - c \right] = \left(\frac{a-c}{2} \right) \left(\frac{a-c}{2} \right) = \frac{(a-c)^{2}}{4}.$$

▶ $\pi^C > \pi_1^D + \pi_2^D$: The integrated system is more **efficient**.

- ▶ Through appropriate profit splitting, both firm earns more.
 - ▶ Integration can result in a **win-win** solution for firms!
- However, under monopoly the aggregate quantity is lower and the price is higher. Consumers benefits from firms' competition.

The two firms' prisoners' dilemma

- ▶ Now we know the two firms should together produce $Q = \frac{a-c}{2}$.
- What if we suggest them to produce $q'_1 = q'_2 = \frac{a-c}{4}$?
- ▶ This maximizes the total profit but is **not** a Nash equilibrium:
 - If he chooses $q' = \frac{a-c}{4}$, I will move to

$$q'' = \frac{1}{2}(a - q' - c) = \frac{3(a - c)}{8}.$$

▶ So both firms will have incentives to unilaterally deviate.

▶ These two firms are engaged in a prisoners' dilemma!

Equilibrium concepts 00000

Road map

▶ Static games.

- ▶ Dynamic games.
 - **Backward** induction.
 - Pricing in a supply chain.

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Dynamic games

▶ Recall the game "BoS":

Player 2 Player 1 $\begin{array}{c|c} Player 2 \\ \hline B & S \\ \hline B & 2,1 & 0,0 \\ \hline S & 0,0 & 1,2 \end{array}$

- ▶ What if the two players make decisions **sequentially** rather than simultaneously?
 - ▶ What will they do in equilibrium?
 - ▶ How do their payoffs change?
 - ▶ Is it better to be the **leader** or the **follower**?

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Game tree for dynamic games

- Suppose player 1 moves first.
- Instead of a game matrix, the game can now be described by a game tree.
 - At each internal node, the label shows who is making a decision.
 - At each link, the label shows an action.
 - At each leaf, the numbers show the payoffs.
- ▶ The games is played from the root to leaves.



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Optimal strategies

- ▶ How should player 1 move?
- She must **predict** how player 2 will response:
 - ▶ If B has been chosen, choose B.
 - ▶ If S has been chosen, choose S.
- ► This is player 2's **best response**.
- Player 1 can now make her decision:
 - ▶ If I choose B, I will end up with 2.
 - ▶ If I choose S, I will end up with 1.
- ▶ So player 1 will choose B.
- ► An equilibrium outcome is a "path" goes from the root to a leaf.
 - ▶ In equilibrium, they play (B, B).

	2	В	2, 1
1	В	S	~ 0, 0
	S 2	В	_0,0
	×	S	>1, 2

Sequential moves vs. simultaneous moves

- ▶ In the static version, there are two pure-strategy Nash equilibria:
 - \triangleright (B, B) and (S, S).
- When the game is played dynamically with player 1 moves first, there is only one equilibrium outcome:

▶ (B, B).

- ▶ Their **equilibrium behaviors** change. Is it always the case?
- ▶ Being the leader is beneficial. Is it always the case?

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Dynamic matching pennies

Suppose the game "matching pennies" is played dynamically:



- ▶ What is the equilibrium outcome?
- ▶ There are multiple possible outcomes.
- Being the leader **hurts** player 1.



Backward induction

- ▶ In the previous two examples, there are a leader and a follower.
- Before the leader can make her decision, she must anticipate what the follower will do.
- ▶ When there are multiple **stages** in a dynamic game, we generally analyze those decision problems **from the last stage**.
 - The second last stage problem can be solved by having the last stage behavior in mind.
 - ▶ Then the third last stage, the fourth last stage, ...
- ▶ In general, we move **backwards** until the first stage problem is solved.
- ▶ This solution concept is called **backward induction**.

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Road map

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Pricing in a supply chain

▶ There is a manufacturer and a retailer in a supply chain.



- ▶ The manufacturer supplies to the retailer, who then sells to consumers.
- The manufacturer sets the wholesale price w and then the retailer sets the retail price r.
- ▶ The demand is D(r) = A Br, where A and B are known constants.
- The unit production cost is C, a known constant.
- Each of them wants to maximize her or his profit.

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Pricing in a supply chain (illustrative)



- Let's assume A = B = 1 and C = 0 for a while.
- Let's apply backward induction to **solve** this game.
- ▶ For the retailer, the wholesale price is **given**. He solves

$$\max_{r\geq 0} (r-w)(1-r).$$

• The optimal solution (best response) is $r^*(w) \equiv \frac{w+1}{2}$.

Pricing in a supply chain (illustrative)



▶ The manufacturer **predicts** the retailer's decision:

- Given her offer w, the retail price will be $r^*(w) \equiv \frac{w+1}{2}$.
- ▶ More importantly, the **order quantity** (which is the demand) will be

$$1 - r^*(w) = 1 - \frac{w+1}{2} = \frac{1-w}{2}$$

▶ The manufacturer's solves

$$\max_{w \ge 0} \ w\left(\frac{1-w}{2}\right).$$

• The optimal solution is $w^* = \frac{1}{2}$.

Pricing in a supply chain (illustrative)



▶ As the manufacturer offers $w^* = \frac{1}{2}$, the resulting retail price is

$$r^* \equiv r^*(w^*) = \frac{w^* + 1}{2} = \frac{3}{4} > \frac{1}{2} = w^*.$$

A common practice called **markup**.

- The sales volume is $D(r^*) = 1 r^* = \frac{1}{4}$.
- The retailer earns $(r^* w^*)D(r^*) = (\frac{1}{4})(\frac{1}{4}) = \frac{1}{16}$.
- The manufacturer earns $w^*D(r^*) = (\frac{1}{2})(\frac{1}{4}) = \frac{1}{8}$.
- In total, they earn $\frac{1}{16} + \frac{1}{8} = \frac{3}{16}$.

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Pricing in a supply chain (general)

▶ For the retailer, the wholesale price is given. He solves

$$\max_{r\geq 0} (r-w)(A-Br)$$

▶ The optimal solution is $r^*(w) \equiv \frac{Bw+A}{2B}$.

▶ The manufacturer predicts the retailer's decision:

- Given her offer w, the retail price will be $r^*(w) \equiv \frac{Bw+A}{2B}$.
- More importantly, the order quantity (which is the demand) will be $A Br^*(w) = A \frac{Bw+A}{2} = \frac{A-Bw}{2}$.

▶ The manufacturer's problem:

$$\max_{w \ge 0} (w - C) \left(\frac{A - Bw}{2}\right)$$

• The optimal solution is $w^* = \frac{BC+A}{2B}$.

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Pricing in a supply chain (general)

- ► As the manufacturer offers $w^* = \frac{BC+A}{2B}$, the resulting retail price is $r^* \equiv r^*(w^*) = \frac{Bw^*+A}{2B} = \frac{BC+3A}{4B}$.
- ▶ The sales volume is $D(r^*) = A Br^* = \frac{A BC}{4}$.
- ► The retailer earns $(r^* w^*)D(r^*) = (\frac{A-BC}{4B})(\frac{A-BC}{4}) = \frac{(A-BC)^2}{16B}$.
- ▶ The manufacturer earns $(w^* C)D(r^*) = (\frac{A BC}{2B})(\frac{A BC}{4}) = \frac{(A BC)^2}{8B}$.
- ▶ In total, they earn $\frac{(A-BC)^2}{16B} + \frac{(A-BC)^2}{8B} = \frac{3(A-BC)^2}{16B}$.

Pricing in a cooperative supply chain

- Suppose the two firms are **cooperative**.
- ▶ They decide the wholesale and retail prices together.
- ▶ Is there a way to allow both players to be **better off**?
- Consider the following proposal:
 - Let's set $w^{\text{FB}} = C = 0$ and $r^{\text{FB}} = \frac{1}{2}$ (FB: **first best**).
 - The sales volume is

$$D(r^{\rm FB}) = 1 - \frac{1}{2} = \frac{1}{2}.$$

The total profit is

$$r^{FB}D(r^{FB}) = \frac{1}{4}.$$

▶ This is larger than $\frac{3}{16}$, the total profit generated under decentralization.

▶ How to split the pie to get a **win-win** situation?

Dynamic games with embedded static games

- ▶ We may have dynamic games with **embedded static games**.
- Consider the following game:
 - A manufacturer setting a wholesale price w and then two retailers each setting an order quantity q_i .
 - Retailer *i*'s utility function is $q_i[a (q_1 + q_2) w]$.
 - The manufacturer's utility function is $(w c)(q_1 + q_2)$.
- ► Backward induction:
 - ▶ In stage 2, the Nash equilibrium is $(q_1^*, q_2^*) = (\frac{a-w}{3}, \frac{a-w}{3})$.
 - In stage 1, the equilibrium wholesale price is $w^* = \frac{a+c}{2}$.
- ▶ Recall the prisoners' dilemma:
 - The game designer should design the game (by determining the penalties) to induce the prisoners to confess!

Road map

- ▶ Static games: Nash equilibrium.
- ▶ Dynamic games: Backward induction.
- ▶ Price of anarchy.
- ▶ Four fundamental equilibrium concepts.

Measuring efficiency loss

- ▶ In many cases, we want to measure **efficiency loss** due to decentralization.
 - ▶ In general there may be **multiple equilibria**. Which one to choose?
 - Similar to the "approximation ratio" in an approximation algorithm or the "competitive ratio" in an online algorithm, let's choose the worst one!
- Consider a game with a set of players N. Player *i*'s strategy set is S_i and utility function is $u_i : S \to \mathbb{R}^+$, where $S = S_1 \times \cdots \times S_n$.
- Let a measure of efficiency of each outcome be $z: S \to \mathbb{R}^+$.
 - Example 1 (sum of all utilities): $z(s) = \sum_{i \in N} u_i(s)$.
 - Example 2 (lowest utility): $z(s) = \min_{i \in N} u_i(s)$.
- ▶ Let $S_e \subseteq S$ be the set of strategy profiles that may exist in equilibrium.

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Suppose that we want to maximize $z(\cdot)$.

Definition 2 (Price of anarchy)

The **price of anarchy** (PoA) of a game $(N, \{S_i\}, \{u_i\})$ with the objective of maximizing the efficiency measure $z(\cdot)$ is

$$\operatorname{PoA} = \frac{\max_{s \in S} z(s)}{\min_{s \in S_e} z(s)}.$$

- The price of anarchy is formed by a best centralized outcome and a worst decentralized outcome.
- ▶ It measures how the efficiency of a system degrades due to decentralized decision making in the worst case.
- ▶ If we want to minimize $z(\cdot)$, we have PoA = $\frac{\min_{s \in S_e} z(s)}{\max_{s \in S} z(s)}$.

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Price of stability

► The **price of stability** (PoS) considers the **best** decentralized outcome and is defined as

$$PoA = \frac{\max_{s \in S} z(s)}{\max_{s \in S_e} z(s)}.$$

- By definition, we know that $1 \leq PoS \leq PoA$.
 - ▶ The efficiency loss due to decentralization is between PoS and PoA.

Example 1: prisoners' dilemma

Recall our prisoners' dilemma

- Let the efficiency measure be the sum of the absolute value of utilities. We aim to minimize this.
- The best centralized outcome results in z(D, D) = 2.
- The worst (actually unique) decentralized outcome results in z(C, C) = 12.
- Both the PoA and PoS are $\frac{12}{2} = 6$.

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Example 2: Cournot competition

- ▶ Recall the Cournot competition.
 - ▶ $q_1^* = q_2^* = \frac{a-c}{3}$.
 - Each of them earns $\frac{(a-c)^2}{9}$.
- ▶ Let the efficiency measure be the **sum of the two firm's profits**. We aim to maximize this.
- ▶ The best centralized outcome results in $z(\frac{a-c}{4}, \frac{a-c}{4}) = \frac{(a-c)^2}{4}$.
- ► The worst (actually unique) decentralized outcome results in $z(\frac{a-c}{3}, \frac{a-c}{3}) = \frac{2(a-c)^2}{9}$.
- ▶ Both the PoA and PoS are

$$\frac{(a-c)^2/4}{2(a-c)^2/9} = \frac{9}{8}.$$

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Example 3: BoS

Recall the BoS. Suppose that the game is now

Player 2
Player 1
$$\begin{array}{c|c} Player 2 \\ \hline B & S \\ \hline B & 3,1 & 0,0 \\ \hline S & 0,0 & 1,2 \end{array}$$

- Let the efficiency measure be the sum of utilities. We aim to maximize this.
- The best centralized outcome results in z(B, B) = 4.
- ▶ The worst decentralized outcome results in z(S, S) = 3. The PoA is $\frac{4}{3}$. Note that it is not $\frac{4}{0}$!
- ▶ The best decentralized outcome results in z(B, B) = 4. The PoS is 1.

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Example 4: supply chain pricing

- ▶ Recall the supply chain pricing game $(w^* = \frac{1}{2}, r^* = \frac{1}{4})$.
- ► Let's maximize the **minimum of the two firm's profits**.
- ▶ The worst (actually unique) decentralized outcome results in

$$z\left(\frac{1}{2},\frac{1}{4}\right) = \min\left\{\frac{1}{8},\frac{1}{16}\right\} = \frac{1}{16}.$$

- ▶ What is the best centralized outcome?
 - Together they may generate up to $\frac{1}{4}$ as the total profit.
 - ► To maximize the minimum profit, they should split the total profit equally to make each of them earn ¹/₈.

▶ Both the PoA and PoS are

$$\frac{1/8}{1/16} = 2$$

Road map

- ▶ Static games: Nash equilibrium.
- ▶ Dynamic games: Backward induction.
- ▶ Price of anarchy.
- ► Four fundamental equilibrium concepts.

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Information

- We have introduced static games and dynamic games under **complete information**.
 - ▶ All players know the others' utility functions.
 - ▶ All players know that all players know the others' utility functions.
 - ▶ All players know that all players know that all players know that the others' utility functions.
 - And so on.
- ▶ There are also games with **incomplete information**.
 - Typically, there is at least one player that does not know at least one another player's utility function.
 - ▶ E.g., auction.

Four fundamental equilibrium concepts

▶ We have four types of games, each with a fundamental equilibrium concept:

	Complete information	Incomplete information	
Static games	Nash	Baysian Nash	
Dynamic games	Subgame perfect	Perfect Baysian	

▶ To understand them, please prepare:

- Calculus.
- Probability.
- ▶ A genius' brain, or a beautiful mind.

Equilibrium concepts 00000

Further learning materials



商管研究中的賽局分析 (一):通路選擇、合 約制定與共享經濟 (Game Theoretic Analysis for Business Research (1)) Available now

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Week	Торіс	Lecture	Video	Pre-lecture Problem
1	Review of optimization	Slides	Video	N/A
2	Review of game theory	Slides	Playlist	Problems
3	Channel selection under competition	Slides	Playlist	Problems
4	Channel coordination with returns	Slides	Playlist	Problems
5	No class: National holiday	N/A	N/A	N/A
6	Retail and delivery platforms	Handout	N/A	Problems
7	The screening theory	Slides	Playlist	Problems
8	Endogenous adverse selection	Slides	Playlist	Problems
9	The signaling theory	Slides, Handout	Playlist	Problems
10	Signaling quality through specialization	Slides	Playlist	Problems

http://www.im.ntu.edu.tw/~lckung/ courses/public/IE_English/

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