How Good is a Two-Party Election Game?

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Joint work with

Chi-Jen Lu and Po-An Chen

Theoretical Computer Science (2021)

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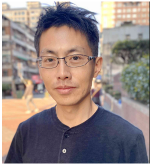

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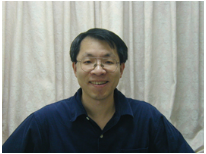

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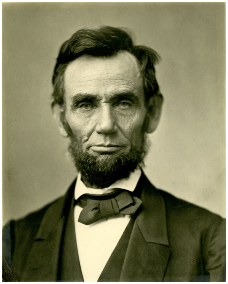
Outline

Outline

The Inspiration



"[...] and that government of the people, by the people, for the people, shall not perish from the earth."

— Abraham Lincoln, 1863.

- Cheng *et al*. Of the People: Voting is more effective with representative candidates. (*EC'17*).

Motivations (I): Why The Two-Party System?

"The simple-majority single-ballot system favours the two-party system."
— Maurice Duverger, 1964.

Motivations (II): Social Choice Rules

Example:

- Each voter provides an ordinal ranking of the candidates,
- Aggregate these rankings to produce either a single winner or a consensus ranking of all (or some) candidates.

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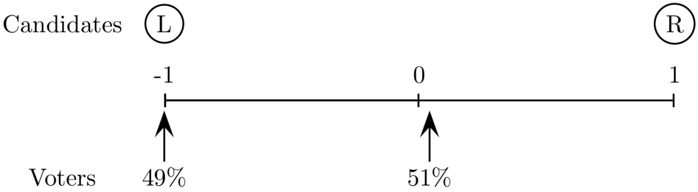
Gibbard–Satterthwaite Theorem (1973)

Given a deterministic electoral system that choose a single winner. For every voting rule, one of the following three things must hold:

- The rule is dictatorial.
- The rule limits the possible outcomes to two alternatives only.
- The rule is susceptible to tactical voting.

Motivations (III): Distortion of Social Choice Rules



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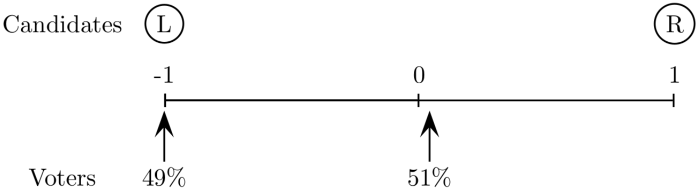

- The average distance from the population to candidate L: ≈ 0.5.
- The average distance from the population to candidate R: ≈ 1.5.
- But R will be elected as the winner in the election.

Issues of Previous Studies

- Voters' behavior on a micro-level.
  - Voters are strategic;
  - Voters have different preferences for the candidates.
  - Various election rules result in different winner(s).

    ⋮

Our Focus

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  - Parties are players;
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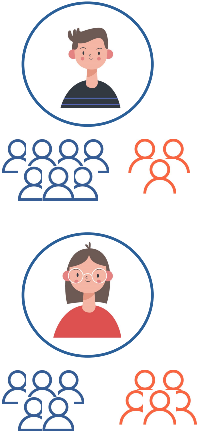
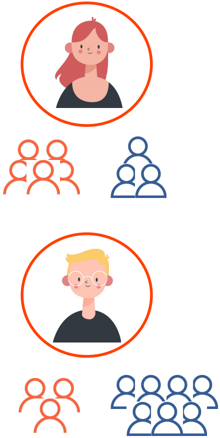
  - The point is:
    - Who is more likely to win the election campaign and how likely is it?
    - Is the game stable in some sense?
    - What's the price for stability which resembles "the distortion"?

Party A



Party B



Party A

√

Winning prob.=0.6

Expected utility for A:
0.6*7+0.4*3 = 5.4

$u(A_1) = 7 + 3 = 10$

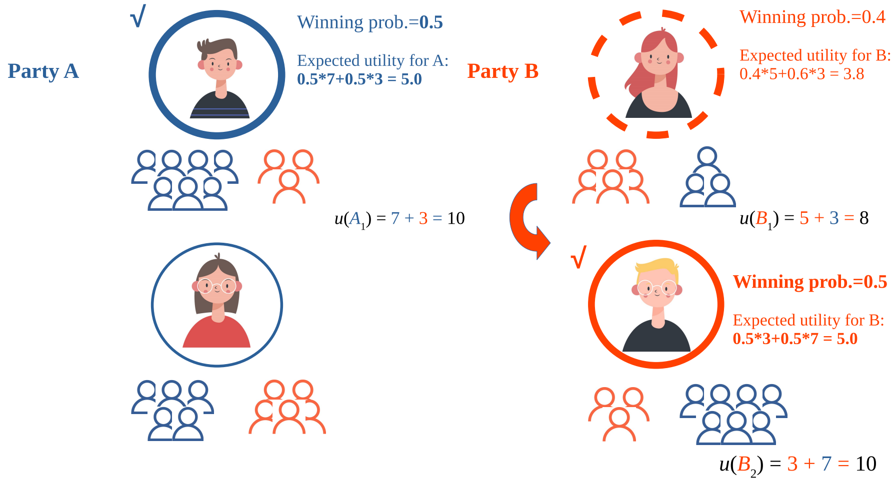
Party B

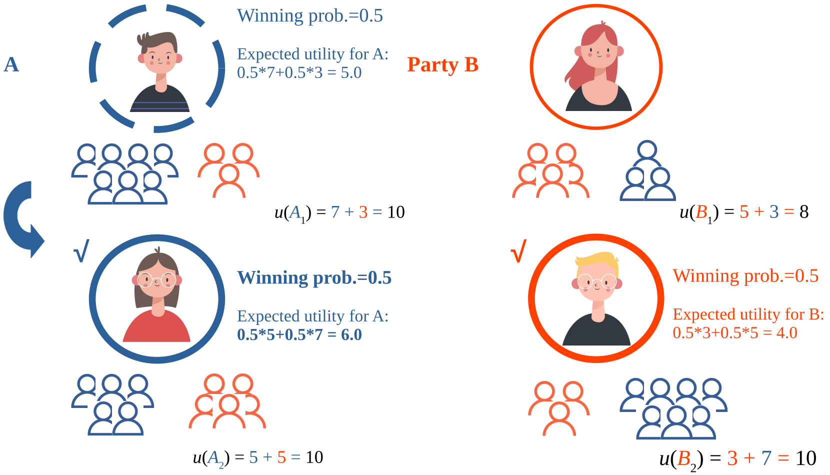
√

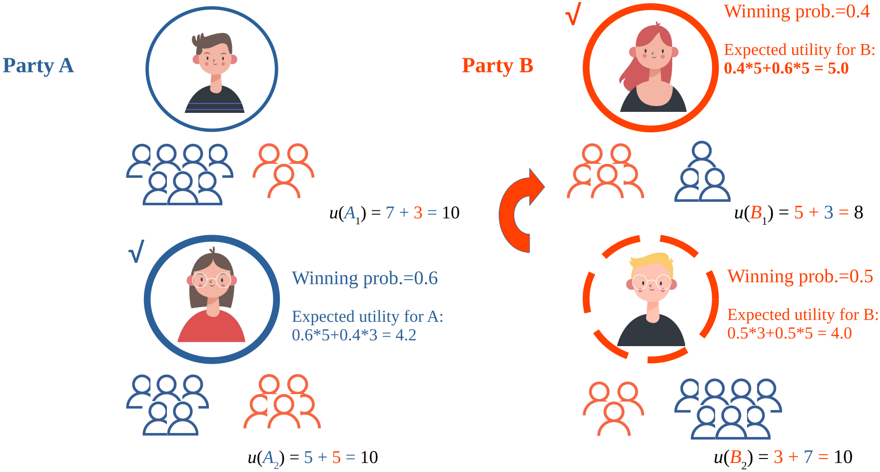
Winning prob.=0.4

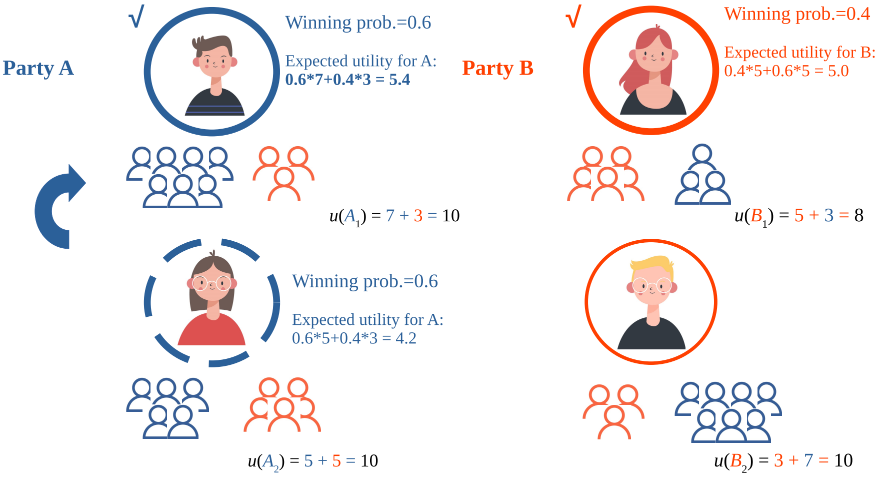
Expected utility for B:
0.4*5+0.6*3 = 3.8

$u(B_1) = 5 + 3 = 8$


Party A
√
Winning prob.=**0.5**
Expected utility for A:
0.5*7+0.5*3 = 5.0
$u(A_1) = 7 + 3 = 10$
Party B
Winning prob.=0.4
Expected utility for B:
0.4*5+0.6*3 = 3.8
$u(B_1) = 5 + 3 = 8$
√
Winning prob.=0.5
Expected utility for B:
0.5*3+0.5*7 = 5.0
$u(B_2) = 3 + 7 = 10$


Party A
Winning prob.=0.5
Expected utility for A:
0.5*7+0.5*3 = 5.0
Party B
u(A1) = 7 + 3 = 10
u(B1) = 5 + 3 = 8
√
Winning prob.=0.5
Expected utility for A:
0.5*5+0.5*7 = 6.0
√
Winning prob.=0.5
Expected utility for B:
0.5*3+0.5*5 = 4.0
u(A2) = 5 + 5 = 10
u(B2) = 3 + 7 = 10


Party A
$u(A_1) = 7 + 3 = 10$
√
Winning prob.=0.6
Expected utility for A:
0.6*5+0.4*3 = 4.2
$u(A_2) = 5 + 5 = 10$
Party B
√
Winning prob.=0.4
Expected utility for B:
0.4*5+0.6*5 = 5.0
$u(B_1) = 5 + 3 = 8$
Winning prob.=0.5
Expected utility for B:
0.5*3+0.5*5 = 4.0
$u(B_2) = 3 + 7 = 10$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$u(A_2) = 5 + 5 = 10$
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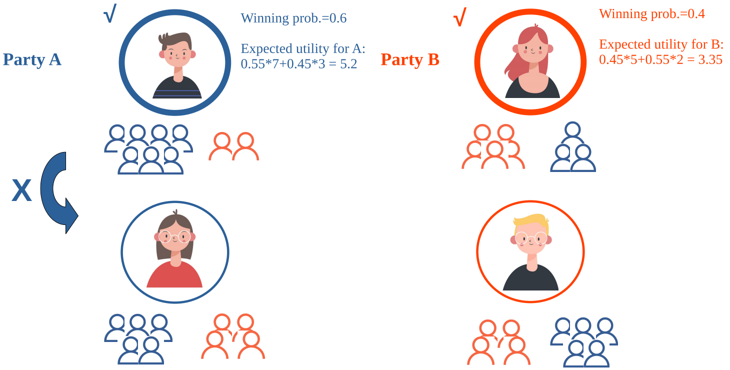
Concept of Stability: Pure Nash Equilibrium

- Each party's strategy: candidate nomination.
- Pure Nash equilibrium (PNE): Neither party A nor B wants to deviate (i.e., change) from their strategy (i.e., nomination) unilaterally.

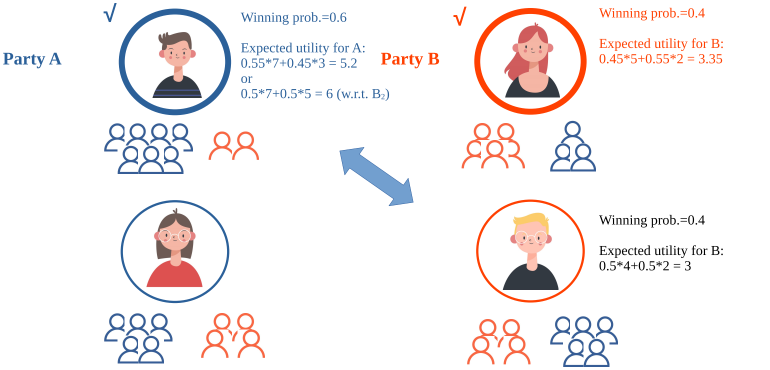
An instance with a PNE.

An instance with a PNE (expected social utility: 8.55).



A Kind of Inefficiency Measure: The Price of Anarchy

An instance with a PNE (expected social utility: 8.55, optimum: 9).



- The price of anarchy (POA): $\frac{9}{8.55} \approx 1.05$.

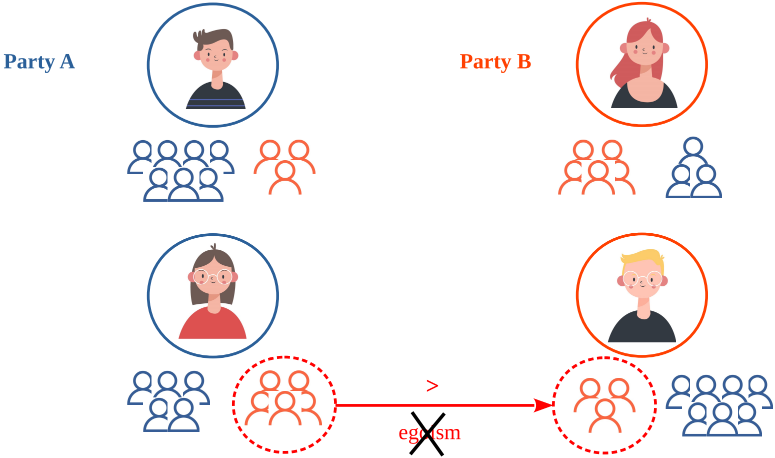
Outline

Two-Party Election Game: Formal Setting

- Party A: m candidates $A_1, A_2, \ldots, A_m$.
  Party B: n candidates $B_1, B_2, \ldots, B_n$.
- A_i: brings utility $u(A_i) = u_A(A_i) + u_B(A_i) \in [0, b]$,
  B_j: brings utility $u(B_j) = u_A(B_j) + u_B(B_j) \in [0, b]$, for some $b \geq 1$.
  - $u_A(A_1) \geq u_A(A_2) \geq \ldots \geq u_A(A_m)$, $u_B(B_1) \geq u_B(B_2) \geq \ldots \geq u_B(B_n)$
- $p_{i,j}$: $\Pr[A_i \text{ wins over } B_j]$.
- Expected utilities:

$$
\begin{aligned}
a_{i,j} &= p_{i,j} u_A(A_i) + (1 - p_{i,j}) u_A(B_j) \\
b_{i,j} &= (1 - p_{i,j}) u_B(B_j) + p_{i,j} u_B(A_i).
\end{aligned}
$$

Egoism (Selfishness)



Two-Party Election Game: Formal Setting (contd.)

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  - $u_A(A_1) \geq u_A(A_2) \geq \ldots \geq u_A(A_m)$, $u_B(B_1) \geq u_B(B_2) \geq \ldots \geq u_B(B_n)$
- $p_{i,j}$: $\Pr[A_i \text{ wins over } B_j]$.
- Expected utilities:

$$\begin{aligned} a_{i,j} &= p_{i,j} u_A(A_i) + (1 - p_{i,j}) u_A(B_j) \\ b_{i,j} &= (1 - p_{i,j}) u_B(B_j) + p_{i,j} u_B(A_i). \end{aligned}$$

- egoistic: $u_A(A_i) > u_A(B_j)$ and $u_B(B_j) > u_B(A_i)$ for all $i \in [m], j \in [n]$.

Two-Party Election Game: Formal Setting (contd.)

- Three models on $p_{i,j}$:
  - Bradley-Terry (Naïve): $p_{i,j} := u(A_i)/(u(A_i) + u(B_j))$
    - Linear dependency on the two social utilities.
    - Intuitive.
  - Linear link: $p_{i,j} := (1 + (u(A_i) - u(B_j))/b)/2$.
    - Linear on the difference between the two social utilities.
    - Dueling bandit setting.
  - Softmax: $p_{i,j} := e^{u(A_i)/b}/(e^{u(A_i)/b} + e^{u(B_j)/b})$
    - Bivariate nonlinear rational function of the two social utilities.
    - Extensively used in machine learning.

Two-Party Election Game: Formal Setting (contd.)

- Three models on $p_{i,j}$:
  - Bradley-Terry (Naïve): $p_{i,j} := 1/(1 + u(B_j)/u(A_i)) \in [0, 1]$.
    - Linear dependency on the ratio of the two social utilities.
    - Intuitive.
  - Linear link: $p_{i,j} := (1 + (u(A_i) - u(B_j))/R)/2 \in [0, 1]$.
    - Linear on the difference between the two social utilities.
    - Dueling bandit setting.
  - Softmax (logistic): $p_{i,j} := 1/(1 + e^{(u(B_j) - u(A_i))/R}) \in \left[\frac{1}{1+e}, \frac{e}{1+e}\right]$.
    - Non-linear (exponential) dependency on the difference between the two social utilities.
    - Extensively used in machine learning.

Two-Party Election Game: Formal Setting (contd.)

- The social welfare of state (i, j):
  $SU_{i,j} = a_{i,j} + b_{i,j}$.
- (i, j) is a PNE if $a_{i',j} \leq a_{i,j}$ for any $i' \neq i$ and $b_{i,j'} \leq b_{i,j}$ for any $j' \neq j$.

Two-Party Election Game: Formal Setting (contd.)

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- (i, j) is a PNE if $a_{i',j} \leq a_{i,j}$ for any $i' \neq i$ and $b_{i,j'} \leq b_{i,j}$ for any $j' \neq j$.
- The PoA of the game:

$$\frac{SU_{i^*,j^*}}{SU_{\hat{i},\hat{j}}} = \frac{a_{i^*,j^*} + b_{i^*,j^*}}{a_{\hat{i},\hat{j}} + b_{\hat{i},\hat{j}}},$$

  - $(i^*, j^*) = \arg\max_{(i,j)\in[m]\times[n]}(a_{i,j} + b_{i,j})$: **the optimal state**.
  - $(\hat{i}, \hat{j}) = \arg\min_{\substack{(i,j)\in[m]\times[n] \\ (i,j) \text{ is a PNE}}}(a_{i,j} + b_{i,j})$: the PNE with **the worst** social welfare.

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Preliminary Inspections for the PNE

Focus on $m = n = 2$ first.

- First try: by human brains and human eyes.

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  - Difficult. ☹

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Focus on $m = n = 2$ first.

- First try: by human brains and human eyes.
  - Difficult. ☹
- Random sampling: ☺
  - Sampling the values of $u_A(A_i), u_B(A_i), u_A(B_j), u_B(B_j)$ for each i, j and the constant b for hundreds of millions times.
  - Experiments for the three winning probability models.

Example: No PNE in the Bradley-Terry Model

$m = n = 2$, $b = 100$ (left: egoistic, right: non-egoistic).

Left (egoistic):

$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
A		B	
91	0	11	1
90	8	10	20

	B_1	B_2
A_1	80.51, 1.28	73.84, 2.17
A_2	80.29, 8.32	74.02, 8.23

Right (non-egoistic):

$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
A		B	
44	10	37	17
39	55	10	5

	B_1	B_2
A_1	30.50, 23.50	35.52, 10.00
A_2	30.97, 48.43	34.32, 48.81

Example: No PNE in the Linear-Link Model (Non-Egoism)

$m = n = 2, \ b = 100.$

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
50	10	10	90
5	20	5	20

	B_1		B_2	
A_1	78,	10	40.25,	8.375
A_2	79.375,	11.25	12.5,	12.5

Non-Egoistic Games Seem to Be Bad ☹

- ⋆ In our experiments, **EVERY** egoistic game instance in the linear-link/softmax model has a PNE!

Non-Egoistic Games Seem to Be Bad ☹

- ⋆ In our experiments, **EVERY** egoistic game instance in the linear-link/softmax model has a PNE!
- The following discussions on equilibrium existence consider only egoistic games.

The Dominating-Strategy Equilibrium

Lemma (The Dominating-Strategy Equilibrium)

- *If* $u(A_1) > u(A_i)$ *for each* $i \in [n] \setminus \{1\}$*, then* $(1, j^{\#})$ *is a PNE for* $j^{\#} = \arg\max_{j \in [m]} b_{1,j}$.
- *If* $u(B_1) > u(B_j)$ *for each* $j \in [m] \setminus \{1\}$*, then* $(i^{\#}, 1)$ *is a PNE for* $i^{\#} = \arg\max_{i \in [n]} a_{i,1}$.

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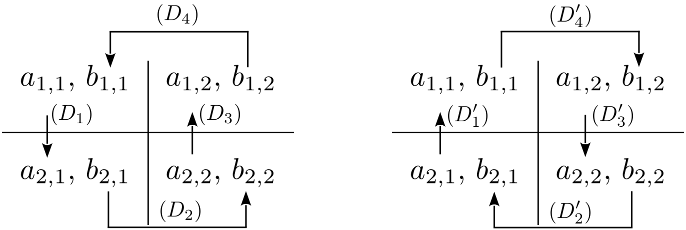
- Hence, the puzzles come from the other cases...

No PNE ⇔ Cycles of Deviations



Deviations → Inequalities

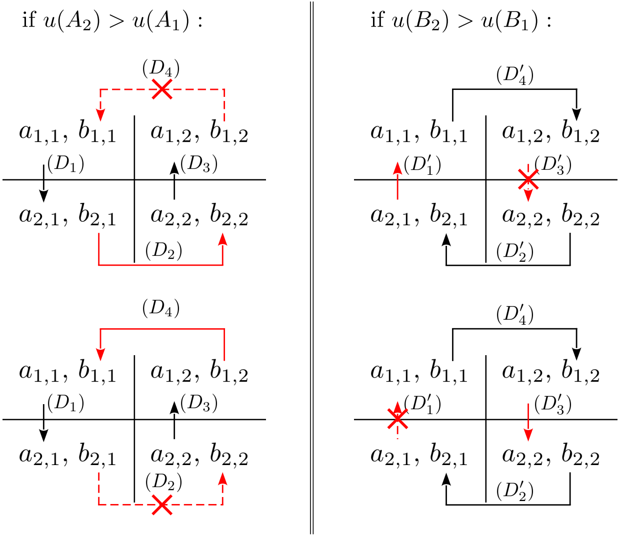
$$\begin{aligned}\Delta(D_1) &= -\Delta(D_1') = a_{2,1} - a_{1,1} \\ &= p_{2,1}u_A(A_2) + (1-p_{2,1})u_A(B_1) \\ &\quad -(p_{1,1}u_A(A_1) + (1-p_{1,1})u_A(B_1)) \\ &= -p_{1,1}(u_A(A_1) - u_A(A_2)) \\ &\quad +(p_{2,1} - p_{1,1})(u_A(A_2) - u_A(B_1)).\end{aligned}$$

$$\begin{aligned}\Delta(D_3) &= -\Delta(D_3') = a_{1,2} - a_{2,2} \\ &= p_{1,2}u_A(A_1) + (1-p_{1,2})u_A(B_2) \\ &\quad -(p_{2,2}u_A(A_2) + (1-p_{2,2})u_A(B_2)) \\ &= p_{1,2}(u_A(A_1) - u_A(A_2)) \\ &\quad +(p_{1,2} - p_{2,2})(u_A(A_2) - u_A(B_2)).\end{aligned}$$

$$\begin{aligned}\Delta(D_2) &= -\Delta(D_2') = b_{2,2} - b_{2,1} \\ &= (1-p_{2,2})u_B(B_2) + p_{2,2}u_B(A_2) \\ &\quad -((1-p_{2,1})u_B(B_1) + p_{2,1}u_B(A_2)) \\ &= -(1-p_{2,1})(u_B(B_1) - u_B(B_2)) \\ &\quad +(p_{2,1} - p_{2,2})(u_B(B_2) - u_B(A_2)).\end{aligned}$$

$$\begin{aligned}\Delta(D_4) &= -\Delta(D_4') = b_{1,1} - b_{1,2} \\ &= (1-p_{1,1})u_B(B_1) + p_{1,1}u_B(A_1) \\ &\quad -((1-p_{1,2})u_B(B_2) + p_{1,2}u_B(A_1)) \\ &= (1-p_{1,1})(u_B(B_1) - u_B(B_2)) \\ &\quad +(p_{1,2} - p_{1,1})(u_B(B_2) - u_B(A_1)).\end{aligned}$$

The Crucial Lemma



The Crucial Lemma

Lemma (Main Lemma for the Linear-Link & Softmax Models)

Consider the two-party election game in the linear-link/softmax model.

- *If* $u(A_2) > u(A_1)$*, then*
  - $\Delta(D_2) > 0 \Rightarrow \Delta(D_4) < 0$
  - $\Delta(D_4) > 0 \Rightarrow \Delta(D_2) < 0.$
- *If* $u(B_2) > u(B_1)$*, then*
  - $\Delta(D'_1) > 0 \Rightarrow \Delta(D'_3) < 0.$
  - $\Delta(D'_3) > 0 \Rightarrow \Delta(D'_1) < 0.$

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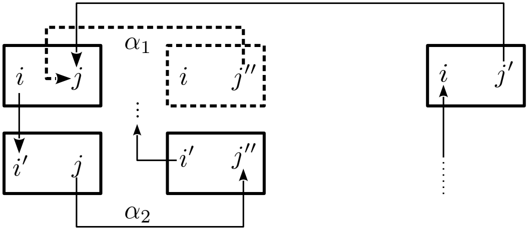
  - $\Delta(D'_1) > 0 \Rightarrow \Delta(D'_3) < 0$.
  - $\Delta(D'_3) > 0 \Rightarrow \Delta(D'_1) < 0$.

Theorem (First Equilibrium Existence Result for $m = n = 2$)

In the linear-link/softmax model with $m = n = 2$, *the two-party election game always has a PNE.* ☺

Outline

What if a party has three or more candidates?



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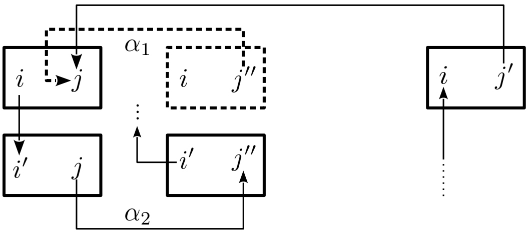

Theorem (Equilibrium Existence Result for $m, n \geq 2$)

The two-party election game with $m \geq 2$ and $n \geq 2$ always has a PNE in the linear-link/softmax model. ☺

Summary of Our Results

	Linear Link	Bradley-Terry	Softmax
PNE w/ egoism	✓	×	✓
PNE w/o egoism	×	×	?#

Outline

Relating PNE to OPT

- i dominates i': $i < i'$ and $u(A_i) > u(A_{i'})$.

Lemma (Property I: PNE and Domination)

- $\exists\ i'$, i' *dominates* i $\Rightarrow (i, j)$ *is not a PNE for any* $j \in [n]$.
- $\exists\ j'$, j' *dominates* j $\Rightarrow (i, j)$ *is not a PNE for any* $i \in [m]$.

Proposition (Property II: Relating a PNE to the OPT State)

Let's say we have

- (i, j)*: a PNE*
- (i^*, j^*)*: the optimal state.*

Then, $u(A_i) + u(B_j) \geq \max\{u(A_{i^*}), u(B_{j^*})\}$.

Illustrating Example: In the Linear-Link Model

For $i \in [m]$, $j \in [n]$,

$$\begin{aligned} SU_{i,j} &= p_{i,j} \cdot u(A_i) + (1 - p_{i,j}) \cdot u(B_j) \\ &= \frac{1 + (u(A_i) - u(B_j))/b}{2} \cdot u(A_i) + \frac{1 - (u(A_i) - u(B_j))/b}{2} \cdot u(B_j) \\ &= \frac{1}{2}(u(A_i) + u(B_j)) + \frac{1}{2b}(u(A_i) - u(B_j))^2 \\ &\geq \frac{1}{2}(u(A_i) + u(B_j)). \end{aligned}$$

and

$$SU_{i,j} = p_{i,j} \cdot u(A_i) + (1 - p_{i,j}) \cdot u(B_j) \leq \max\{u(A_i), u(B_j)\}.$$

Illustrating Example: In the Linear-Link Model (contd.)

Theorem (PoA Bound for Linear-Link)

The two-party election game in the linear link model has PoA $\leq$ 2.

Proof.

(i, j): a PNE; (i^*, j^*): OPT. By the previous Lemma:

$$\begin{cases} i \text{ is not dominated by } i^* \\ j \text{ is not dominated by } j^* \end{cases} \Rightarrow \begin{cases} i \leq i^* \text{ or } u(A_{i^*}) \leq u(A_i) \\ j \leq j^* \text{ or } u(B_{j^*}) \leq u(B_j) \end{cases}$$

- $SU_{i^*,j^*} \leq \max\{u(A_{i^*}), u(B_{j^*})\}$, $\max\{u(A_{i^*}), u(B_{j^*})\} \leq u(A_i) + u(B_j)$.
- $2 \cdot SU_{i,j} \geq u(A_i) + u(B_j)$.

Thus, $SU_{i,j} \geq SU_{i^*,j^*}/2$. □ □

Illustrating Example: In the Linear-Link Model (Lower Bound)

- A tight example (PoA ≈ 2; $\delta \ll \epsilon \ll b$).

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
ϵ	0	ϵ	0
$\epsilon-\delta$	$\epsilon-\delta$	$\epsilon-\delta$	$\epsilon-\delta$

	B_1	B_2
A_1	$\frac{\epsilon}{2}$, $\frac{\epsilon}{2}$	$\epsilon-\frac{\delta}{2}$, $\frac{\epsilon}{2}-\frac{\delta}{2}$
A_2	$\frac{\epsilon}{2}-\frac{\delta}{2}$, $\epsilon-\frac{\delta}{2}$	$\epsilon-\delta$, $\epsilon-\delta$

The PoA of non-egoistic games can be really bad...

Unbounded PoA for Non-Egoistic Games

Linear-Link Model:

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
ϵ	0	ϵ	0
0	b	0	b

	B_1	B_2
A_1	$\frac{\epsilon}{2}, \quad \frac{\epsilon}{2}$	$b - \frac{\epsilon(b-\epsilon)}{2b}, \quad 0$
A_2	$0, \quad b - \frac{\epsilon(b-\epsilon)}{2b}$	$\frac{b}{2}, \quad \frac{b}{2}$

- PoA $= \frac{b}{\epsilon}$.

Unbounded PoA for Non-Egoistic Games

Softmax Model:

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
ϵ	0	ϵ	0
0	b	0	b

	B_1		B_2	
A_1	$\frac{\epsilon e^\epsilon}{e^\epsilon+1}$,	$\frac{\epsilon e^\epsilon}{e^\epsilon+1}$	$\frac{\epsilon e^\epsilon+eb}{e^\epsilon+1}$,	0
A_2	0,	$\frac{\epsilon e^\epsilon+eb}{e^\epsilon+1}$	$\frac{b}{2}$,	$\frac{b}{2}$

- PoA $= \frac{b}{2\epsilon e^\epsilon/(e^\epsilon+1)}$.

Unbounded PoA for Non-Egoistic Games

Bradley-Terry Model:

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
ϵ	0	ϵ	0
0	b	0	b

	B_1		B_2	
A_1	$\frac{\epsilon}{2}$,	$\frac{\epsilon}{2}$	$\frac{\epsilon^2+b^2}{b+\epsilon}$,	0
A_2	0,	$\frac{\epsilon^2+b^2}{b+\epsilon}$	$\frac{b}{2}$,	$\frac{b}{2}$

- PoA $= \frac{b}{\epsilon}$.

Summary of Our Results +(PoA)

	Linear Link	Bradley-Terry	Softmax
PNE w/ egoism	✓	×	✓
PNE w/o egoism	×	×	?#
PoA upper bound w/ egoism	2	2	$1 + e$
PoA lower bound w/ egoism	2	6/5	2
Worst PoA w/o egoism	∞	∞	∞

Outline

Future Work

	Linear Link	Bradley-Terry	Softmax
PNE w/ egoism	✓	×	✓
PNE w/o egoism	×	×	?#
PoA upper bound w/ egoism	2	2	$1+e$
PoA lower bound w/ egoism	2	6/5	2
Worst PoA w/o egoism	∞	∞	∞

Future Work (contd.)

- Three or more parties.
- The correspondence between macro and micro settings.
- PoA w.r.t. NE.

Thank you.

*Special Acknowledgment: Inserted Pictures Were Designed by Freepik.